Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True   **FALSE** The formula for the number of ways to place b indistinguishable balls into u distinguishable urns injectively if $b > u$ is 0 not $\binom{u}{b}$.

> **Solution:** If $b > u$, then $\binom{u}{b} = 0$ so both are valid.

2. **TRUE**   False It is impossible to devise an algorithm to solve the stable marriage problem if men can marry other men (the "roommate problem").

> **Solution:** Zvezda gave an example of a system of preferences such that there is no stable pairing which means that an algorithm cannot give you a stable pairing.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (4 points) How many ways can I buy 200 bubble teas from RareTea for an event if there are 10 different options to choose from?

> **Solution:** There are 200 indistiguishable balls which are the bubble teas, and the boxes are the options. So there are $\binom{200+10-1}{200} = \binom{209}{200}$ ways to do this.

(b) (4 points) The most popular option is original milk tea. How many ways can I do this if I need at least 20 of that option and at least 10 of every other option?

> **Solution:** First I buy 20 of that option and 10 of all the other options. So I've bought 110 bobas and need to buy 90 more out of the 10 options. There are $\binom{90+10-1}{90} = \binom{99}{90}$ ways to do this.

(c) (2 points) Suppose men and women have the preferences $m_1 : w_1 > w_3 > w_2, m_2 : w_2 > w_1 > w_3, m_3 : w_3 > w_2 > w_1$ and $w_1 : m_1 > m_2 > m_3, w_2 : m_2 > m_3 > m_1, w_3 : m_3 > m_2 > m_1$. Is the matching $(m_1, w_1), (m_2, w_3), (m_3, w_2)$ stable?

Solution: This is not stable because m_3 prefers w_3 to w_2 and w_3 prefers m_3 to m_2 which means that both with divorce their partners and get with each other.